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Dynamical SU(8) - A Laboratory for Phase Coexistence

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We consider a model hamiltonian for the coexisting many-electron phenomena of superconductivity, charge density waves and ferro- and anti-ferromagnetism. The spectrum-generating algebra (SGA) for such a model is su(8). We identify all 63 generators of this Lie algebra in physically meaningful bases for which the Cartan elements correspond to symmetries conserved at high temperature. The remaining 56 generators are shown to correspond to the order parameters of the various phases present in the model. A chain of subalgebras is exhibited, and the associated phenomena identified.

The paradigm for our treatment of coexisting many electron phenomena in this note is the classical BCS model of superconductivity[1]. One starts with a general hamiltonian H^{gen} for a system of interacting fermions

$$H^{gen} = \sum_{k,\alpha} \epsilon_k a^{+}_{k\alpha} a_{k\alpha} + \sum_{k_i,\alpha,\beta} V_{k_1 k_2 k_3 k_4} a^{+}_{k_1\alpha} a^{+}_{k_2\beta} a_{k_3\beta} a_{k_4\alpha} \tag{1}$$

where the fermion operators $a_{k\alpha}$ of momentum k and spin α satisfy the canonical anti-commutation relations

$$\{a_{k\alpha}, a_{k'\alpha'}\} = \delta_{kk'}\delta_{\alpha\alpha'} \quad (\alpha,\alpha' = \uparrow \text{ or } \downarrow). \tag{2}$$

Various physical approximations are imposed on (1) to reduce it to manageable form; for superconductivity, the pairing interaction is assumed dominant, leading to the reduced hamiltonian H^{red}

$$H^{red} = \sum_{k,\alpha} \epsilon_k a^{+}_{k\alpha} a_{k\alpha} + \sum_{k,k'} V_{kk'} a^{+}_{k\uparrow} a^{+}_{-k\downarrow} a_{-k'\downarrow} a_{k'\uparrow} \tag{3}$$

We may apply linearization procedure to (3), leading to the final mean-field hamiltonian $H^{mean\text{-}field} = \sum_k H(k)$, where

$$H(k) = \epsilon_k (a^{+}_{k\uparrow} a_{k\uparrow} + a^{+}_{-k\downarrow} a_{-k\downarrow}) + (\Delta^*(k) a_{k\uparrow} a_{-k\downarrow} + \Delta(k) a^{+}_{-k\downarrow} a^{+}_{k\uparrow}) \tag{4}$$

Here $\Delta(k)$ is defined by $\sum V_{kk'} \langle a_{k'\uparrow} a_{-k'\downarrow} \rangle_T$; as the thermodynamic expectation <> at temperature T is with respect to the hamiltonian (4), in which Δ is already present, this is a self-consistent equation for Δ. The dynamical group approach becomes manifest at the mean-field level of equation (4), as H(k) is expressed in terms of fermion pairs, which generate a conpact Lie algebra (in this case $su(2)_{(k)}$). However, algebraic methods play a role at the pairing level (3), since H^{red} may be considered as an element in the enveloping algebra of $\oplus_k su(2)_{(k)}$. It is a relatively straightforward matter to generalize these ideas to more complex interacting electron systems. We start by writing down a model hamiltonian at the mean-field level, determine the associated spectrum generating algebra and, if we wish to pursue questions of self-consistency, move up to the level of the reduced hamiltonian by means of the enveloping algebra. Thus

$$H^{\text{mean-field}} = HKE + HSC + HDW + HFM \tag{5}$$

where
$$HKE = \sum_{k\alpha} \epsilon(k) a^{\dagger}_{k\alpha} a_{k\alpha} \tag{6}$$
$$HSC = \sum_{k} \Delta_0^{*}(k) a_{k\uparrow} a_{-k\downarrow} + h.c \tag{7}$$
$$HDW = \sum_{k,\mu} \gamma_{\mu}(k) a^{\dagger}_{k+Q\alpha} \sigma_{\mu}^{\alpha\beta} a_{k\beta} + h.c. \tag{8}$$
$$HFM = \sum_{k,\alpha,\beta} \underline{f}(k)\, a^{\dagger}_{k\alpha}\, \underline{\sigma}^{\alpha\beta} a_{k\beta} \tag{9}$$

The first two terms on the right-hand side of (5) correspond to the superconducting mean-field hamiltonian (4); we write $\Delta_0{}^*$ with a zero-subscript to emphasize that this refers to singlet superconductivity (spin-zero pairing). The term HDW in (8) represents a charge density wave for $\mu = 0$, and a spin density wave (anti-ferromagnetic) term for $\mu = 1, 2, 3$; summation is over $\mu = 0, 1, 2, 3$ and $\sigma_{\mu} \equiv \frac{1}{2}\tau_{\mu}$ are the usual spin matrices, with $\tau_0 \equiv I$. Here Q is a fixed wave vector, characteristic of the density wave phenomenon; HDW is decoupled into a direct sum of non-interacting terms by assuming that only terms for which $|k| \leq Q$ contribute. Finally, HFM (8) represents a Ferromagnetic field.

The hamiltonian $H^{\text{mean-field}}$ of (5) may be expressed as a direct sum $H^{\text{mean-field}} = \sum_k H(k)$, where $H(k) = \sum_{i,j=1}^{8} m_{ij}(k) X_{ij}(k)$
with $X_{ij}(k) \equiv B_i{}^{+}(k) B_j(k)$, $(i,j = 1, 2, \ldots, 8)$
and $B_i(k) = \{a_{k\uparrow}, a^{\dagger}_{-k\downarrow}, a_{k-Q\uparrow}, a^{\dagger}_{-(k-Q)\downarrow}, a_{k\downarrow}, a^{\dagger}_{-k\uparrow}, a_{k-Q\downarrow}, a^{\dagger}_{-(k-Q)\uparrow}\}$
From the anti-commutation rules $\{B_i(k) B_j{}^{+}(k') = \delta_{ij}\delta_{kk'}$ we have

$$[X_{ij}(k), X_{rs}(k')] = (\delta_{jr} X_{ij}(k) - \delta_{is} X_{rj}(k))\delta_{kk'} \quad (i,j,r,s = 1 \ldots 8). \tag{10}$$

These are the commutation relations for gl(8); because H(k) is hermitian and traceless, and its elements generate the full algebra, the relevant spectrum generating algebra for our model is

$\oplus_k su(8)_{(k)}$. (As the direct sum adds no complication, we shall simply refer to the SGA as su(8).) We have previously noted [2] that in such a picture the Cartan elements X_{ii} (i = 1, ..., 8) (it is more convenient to work in u(8) here) correspond to conservation laws broken by the phase transition from H^{red} to $H^{mean-field}$; while the X_{ij} correspond to order operaters - whose expectations give the order parameters. With this in mind we may classify the 56 order operators as follows: 4 quartets of singlet-triplet superconductivity, 4 quartets of charge-spin density wave, 4 quartets of singlet-triplet anomalous (i.e. number non-conserving and momentum non-conserving) and 8 ferromagnetic. The physical character of these order operators is determined by the commutation properties with the Cartans X_{ii} (Number, Momentum, etc.) and their spin content by commutation with the spin operator $\sum_{k,\alpha,\beta} a^{+}_{k\alpha} \underline{\sigma}^{\alpha\beta} a_{k\beta}$. They may be further distinguished by their discrete (Parity, Time Inversion) transformation properties. Self-consistent equations for the $m_{ij}(k)$ of this model, which correspond to the $\Delta(k)$ of (4), may be obtained from a reduced hamiltonian corresponding to (3). Such a hamiltonian is an element of the enveloping algebra of $\oplus_k su(8)_{(k)}$ which commutes with all the $\sum_{i,j} X_{ii}(k)$ (the conserved quantities); a suitable choice is

$$H^{red} = \sum_{i,j,k,k'} g_{ij}(k,k')X_{ij}(k)X^{+}_{ij}(k') \tag{11}$$

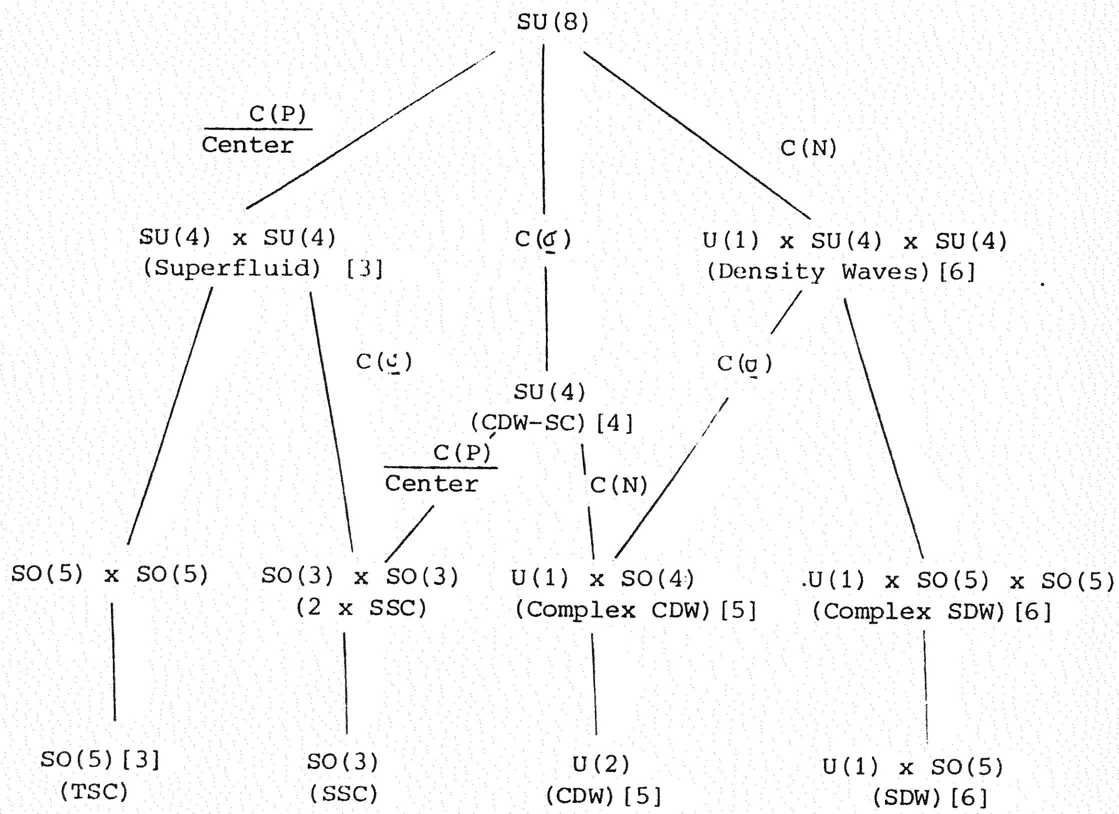
The self-consistent equations have the form

$$m_{ij}(k) = < \sum_{k'} g_{ij}(k,k')X^{+}_{ij}(k') >_T \tag{12}$$

where the thermal average $<>_T$ is taken with respect to $H^{mean-field}$(5).

The su(8) group is rich in subgroups providing a variety of physical submodels; this is why it proves such an excellent laboratory for experimenting with different types of phases. Most - although not all - of these submodels may be obtained by the method of centralizers. Thus the appropriate spectrum generating algebra for a model conserving momentum P is obtained by taking the centralizer of P in su(8) - and factoring out P, since P does not occur in the hamiltonian. By this means one obtains the algebra $C(P)_{su(8)}/P \sim su(4)\oplus su(4)$ for a pair (k and k-Q) of mixed singlet triplet superconductors. Taking the centralizer of spin $\underline{\sigma}$ in this system eventually leads to the usual so(3) BCS model. Similarly, $C_{su(8)}(N)$ is the appropriate algebra for density waves; we do not factor N out since it must occur in the kinetic energy part of the hamiltonian. The accompanying diagram gives one example of such a chain of subalgebras; the references indicate where the corresponding submodels have been treated in more detail.

DIAGRAM   Subgroup Descent from SU(8)



Notation: SSC = Singlet Superconductor
TSC = Triplet Superconductor
CDW = Charge Density Waves
SDW = Spin Density Waves

References

[1] J. Bardeen, L.N. Cooper and J.K. Schrieffer, Phys. Rev. 108, 1175 (1957).

[2] A.I. Solomon, "A Lie Algebraic Approach to Order Parameters" in 'Differential Geometric Methods in Mathematical Physics', S Sternberg (Ed.), page 279 (D. Reidel, 1984).

[3] A. I. Solomon, J. Phys. A14, 2177 (1981).

[4] J. L. Birman and A. I. Solomon, Phys. Rev. Lett. 49, 230 (1982).

[5] A. I. Solomon and J. L. Birman, Phys. Lett. 88A, 413 (1982).

[6] A. I. Solomon and J.L. Birman, Phys. Lett. 104A, 235 (1984).